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LETTER TO THE EDITOR

V. I. Fabrikant, "The stress intensity factor for an external elliptical crack", *Int. J. Solids Structures*, Vol. 23, pp. 465–467 (1987)

We were indeed surprised to find the above-referenced paper which appeared recently in the journal.

What was falsely claimed as "...an error in the well-known formula by Kassir and Sih (1975) related to the stress intensity factor for an external elliptical crack in a three dimensional medium..." turns out to be a mistake on the part of the author. Not knowing that the incorrect result was arrived at in a different coordinate system, he further advises the readers to make "...similar corrections...in the other formulae in Kassir and Sih (1975) treating the stress intensity factor for an elliptical crack under various conditions...." These claims are misleading and will be shown to be incorrect.

Consider the stress intensity factor formula under discussion (Kassir and Sih, 1968)

$$k_1 = \frac{P^{\infty}}{2\pi(ab)^{1/2}}(a^2 \sin^2 \phi + b^2 \cos^2 \phi)^{-1/4} \tag{1}$$

where P^{∞} is an axial force acting at infinity in the z-direction, a and b are, respectively, the major and minor semi-axes of the ellipse and ϕ is the parametric angle of an arbitrary point (A) on the boundary of the ellipse (Fig. 1). The coordinates of point (A) are $x = a \cos \phi$ and $y = b \sin \phi$. In establishing eqn (1), the normal stress distribution due to P^{∞}, i.e.

$$\sigma_z(x, y, 0) = \frac{P^{\infty}}{2\pi ab}\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^{-1/2} \tag{2}$$

is evaluated at a point such as (B) and then expanded asymptotically for small values of the normal distance (r). Point (B) is situated along the normal to the crack periphery and has the coordinates: $x = a \cos \phi - r \cos \beta$, $y = b \sin \phi - r \sin \beta$ where $\beta = \tan^{-1}(a/b \tan \phi)$, as indicated in Fig. 1.

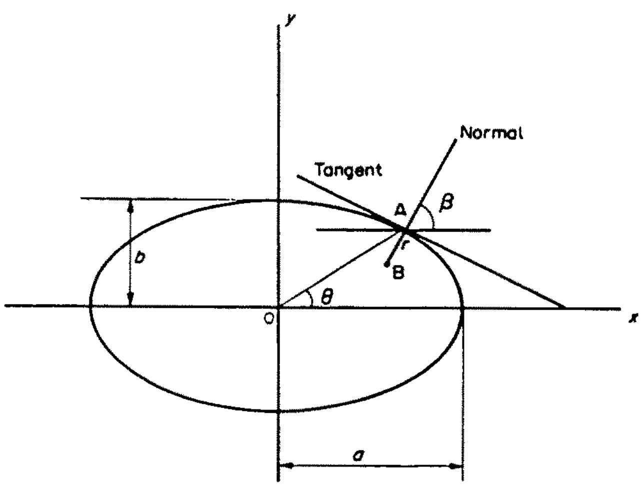

Fig. 1.

The author introduced polar coordinates (ρ, θ) of a point in the elliptical bonded region and determined the stress intensity factor along the polar radius through point (A) in the form

$$k_1 = \frac{P^{\infty}}{2\pi(ab)^{1/2}}(a^2\cos^2\theta + b^2\sin^2\theta)^{-1/4}. \tag{3}$$

The apparent similarity between eqns (1) and (3) has led the author to the conclusion that eqn (1) is incorrect! The symbol ϕ should not be confused with the polar angle θ. In fact, the angles ϕ and θ are related as follows:

$$a\cos\phi = c(\theta)\cos\theta$$
$$b\sin\phi = c(\theta)\sin\theta \tag{4}$$

where $c(\theta) = ab/(a^2\sin^2\theta + b^2\cos^2\theta)^{1/2}$ stands for the distance OA. He further casts eqn (3) along the normal to the crack border and arrives at

$$k_1 = \frac{P^{\infty}}{2\pi(ab)^{1/2}}\left[\frac{a^2\sin^2\theta + b^2\cos^2\theta}{a^4\sin^2\theta + b^4\cos^2\theta}\right]^{1/4} \tag{5}$$

to which he claims as a new formula for the stress intensity factor. Inserting eqns (4) into eqn (5) leads to eqn (1). This completes the proof for the stress intensity factor. Hence, there is nothing wrong with eqn (1) as the author was simply confused with the angles ϕ and θ. His conclusion has no foundation and should be dismissed.

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AUTHOR'S CLOSURE

Mathematics is a powerful tool: it gives us the ability to make almost any formula correct. Here is how. Suppose we have a formula

$$k = F(\theta)$$